techniques $QN^2 \times 10^{-5} = 9.3$, 10.5, 20.9, for $CSOR_b$, GS_b, and J_b, respectively and $QN^2.10^{-5} > 9.3$ for SOR_b due to computational times incurred in determining the optimum value of $\alpha (= 1.1$ in this case).

Testing of the four techniques for other configurations and Mach numbers, subsonic and supersonic, have been performed within the methods of Ref. 1 with essentially equivalent results. Thus, it has been demonstrated that the use of $CSOR_b$ results in material savings in computational times over the use of J_b, GS_b, and SOR_b.

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Prediction of Airfoil Tone Frequencies

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AIRFOILS tested at low Reynolds numbers and moderate incidence are found[1-5] to radiate sound as a discrete tone. In one of those studies,[5] it was claimed that this tone is associated with the development of a Karman vortex street in the turbulent near wake. Thus the tone Strouhal number was taken as 0.281 referenced to vortex street width. Measured 1/3 octave frequency bands for the tone were matched by arbitrarily assuming a vortex street width of 0.6 times the sum of calculated upper-surface and lower-surface turbulent boundary-layer thickness at the trailing edge. If this is the correct physical mechanism, airfoil tone noise should occur not just in model tests but in flyovers of all full-scale aircraft. Instead, such noise is observed only[6] in flyovers of small high-performance sailplanes having airfoils designed for large chordwise extent of laminar flow.

An alternate hypothesis[3] was based on results of tests at larger Reynolds number. In those tests, the tone was found to disappear when Reynolds number was increased such that the pressure surface boundary layer became turbulent. If Strouhal number based on laminar boundary-layer thickness were constant, tone frequency would be predicted to vary with velocity to the 1.5 power and airfoil chord to the -0.5 power. Approximately this velocity dependence was noted experimentally[3,4] at constant chord. Also, a boundary-layer trip wire at 70 to 80% chord of the pressure surface was found in two different test programs[1,3] to eliminate tones that protruded more than 20 db above background.

It has been noted by Tam[7] that the observed[3] growth, saturation, and decay of tone strength with increasing airspeed was typical of an acoustically excited aerodynamic feedback loop. He associated this feedback with oscillations of a laminar near wake. Instead, the fluid dynamic oscillations could be Tollmein-Schlichting instability waves within the pressure-surface laminar boundary layer. As these waves are convected past the trailing edge, they would generate trailing edge noise. The resulting acoustic waves, at the frequency for which Tollmein-Schlichting waves at the trailing edge are strongest, would reinforce boundary-layer oscillations at this frequency. Then the acoustic tone frequency should be identical to the frequency of maximum-amplitude waves within the laminar boundary layer.

When amplitude of boundary-layer oscillations is measured at constant frequency and different streamwise positions, maximum amplitude is found[8] to occur at a Reynolds number given by the right branch of Shen's[9] calculated neutral stability contour. This solution was noted by Schlichting[10] to be the most rigorous solution for Tollmein-Schlichting instabilities. Assume for convenience that the airfoil pressure-surface boundary layer can be approximated by that for a flat plate. The tone frequency f should be $(2\pi)^{-1}$ times the instability angular frequency β. The frequency parameter $\beta\nu/U^2$ was given in Fig. 4 of [3] for flat plates as a function of Reynolds number based on displacement thickness. It is more instructive to express the frequency in terms of reduced frequency based on laminar displacement thickness.

$$f = (2\pi)^{-1} \ (\beta\delta^*/U) \ (U/\delta^*) \tag{1}$$

Expressing displacement thickness δ^* in terms of airfoil chord c,

$$f = (2\pi)^{-1} (1.73)^{-1} (\beta\delta^*/U) U^{3/2} (c\nu)^{-1/2} \tag{2}$$

The reduced frequency $\beta\delta^*/U$ calculated from[9] is given in Fig. 1. It decreases from roughly 0.17 to 0.10 as Reynolds number based on chord is increased from 1×10^5 to $2 \times 10.^6$ It is approximately constant over limited ranges of Reynolds number, yielding the form of the equation given in Ref. 3.

Measured tone frequencies for an NACA 0012 airfoil at Reynolds numbers from 0.3×10^6 to $1.4 \times 10,^6$ taken from Fig. 5 of Ref. 3, are compared in Fig. 2 with those calculated from the previous equation. Predicted frequencies are in good agreement with data. The reduced frequency is roughly equal to 0.12 at an average Reynolds number of $1 \times 10,^6$ yielding the numerical factor given in Ref. 3. Discontinuities occurred when integer multiples of wavelength were equal to the chord. Measured tone frequencies for a markedly different airfoil at Reynolds numbers form 0.1×10^6 to 0.2×10^6 were found[4] to

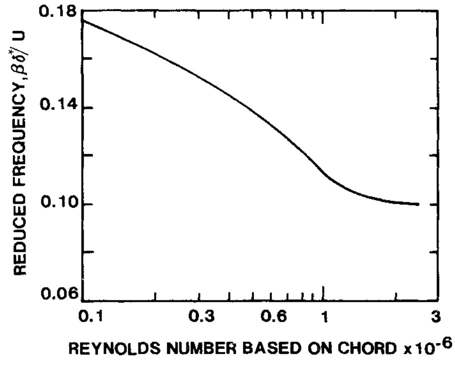

Fig. 1 Calculated reduced frequencies for neutral stability.

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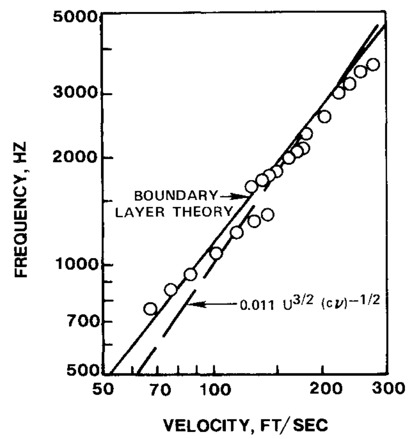

Fig. 2 Comparison with data of Paterson et at.[3]

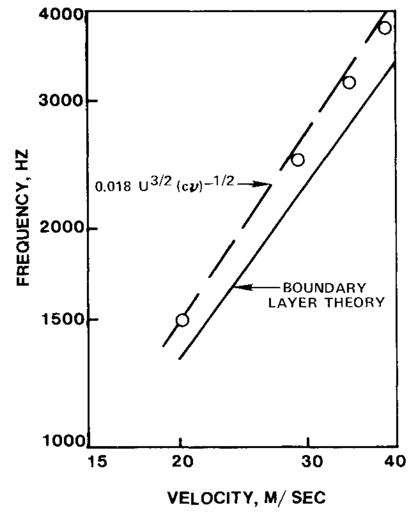

Fig. 3 Comparison with data of Sunyach et al.[4]

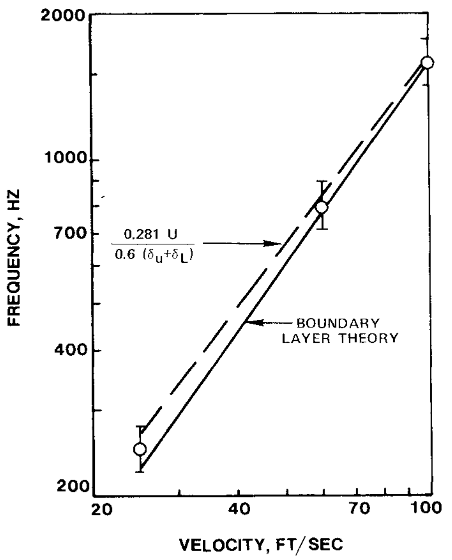

Fig. 4 Comparison with data of Hersh et al.[1]

follow the same trend but with a numerical constant more than 50% larger. As shown in Fig. 3, frequencies calculated by the previous method predict the measured trend of those data but underestimate the data by about 15%. The viscous instability theory also was found[8] to underestimate frequencies for measured peak amplitudes of boundary-layer fluctuations at these low Reynolds numbers. One-third-octave tone frequencies[1,5] for an NACA 0012 airfoil at Reynolds numbers from roughly 0.1×10^6 to 0.3×10^6 are given in Fig. 4. They are as well predicted by this method as by the turbulent wake thickness method[5] that fails to predict data from other investigators. Finally, peak frequencies for uncambered and cambered NACA 65 series airfoils over a range of incidence and Reynolds numbers from 0.1×10^6 to 0.5×10^6, taken from Figs. 14 and 15 of Ref. 2, are compared with calculations in Fig. 5. Flat-plate laminar boundary-layer theory approximately predicts these data.

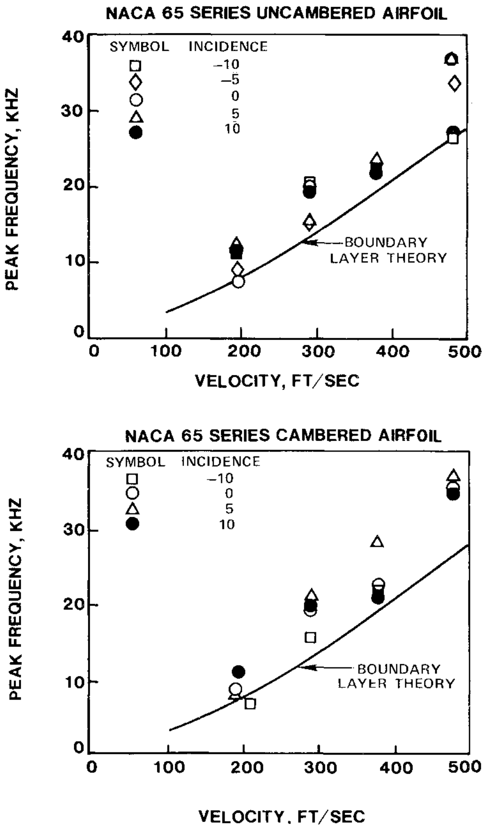

Fig. 5 Comparison with data of Clark.[2]

Conclusions

Measured tone frequencies for airfoils radiating what has been called vortex shedding noise or airfoil wake-generated noise can be predicted directly from laminar boundary-layer instability theory without use of empirical constants. Such noise can be eliminated by tripping the laminar boundary layer at model scale or by increasing the Reynolds number toward full scale.

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Pressure Field of a Vortex Wake in Ground Effect

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THE flowfield produced by the approach of a point vortex pair to a plane boundary was presented years ago by Grobli[1] and can be found in Lamb's, *Hydrodynamics.*[2] Recently the FAA has studied the problem of locating descending vortices generated by aircraft by use of near ground measurements of both velocity and pressure.[3] To correlate the measurements with height and intensity of the vortices a theoretical base is desirable but heretofore the pressure field of the simple vortex field has not been available.

The shed vorticity of a lifting aircraft wing rolls up rather quickly and for an elliptically span-loaded wing the bulk of the vorticity shed from each side is closely centered about the points laterally situated at a distance $\pm\pi s/4$ from the aircraft's centerline; s is the wing semispan distance. Because of the slow variation in the field with distance behind the aircraft, the development of the vortex wake behind the aircraft can be approximated assuming the flow is two-dimensional. Thus for two vortices located at the points $(\pm x_0, y_0)$ with the ground plane at $y=0$, the disturbance velocity potential function can be written

$$\phi=\frac{\Gamma}{2\pi}\left[\tan^{-1}\frac{y-y_0}{x-x_0}+\tan^{-1}\frac{y+y_0}{x+x_0}-\tan^{-1}\frac{y-y_0}{x+x_0}-\tan^{-1}\frac{y+y_0}{x-x_0}\right] \tag{1}$$

where Γ is the circulation of the wing at the centerline and the coordinate system is defined in Fig. 1.

The velocity field follows by differentiation of Eq. (1);

$$u=\frac{\partial\phi}{\partial x}=\frac{\Gamma}{2\pi}\left[\frac{y_0-y}{[(x-x_0)^2+(y-y_0)^2]}-\frac{(y_0+y)}{[(x+x_0)^2+(y+y_0)^2]}+\frac{(y-y_0)}{(x+x_0)^2+(y-y_0)^2}+\frac{(y+y_0)}{(x-x_0)^2+(y+y_0)^2}\right] \tag{2}$$

$$v=\frac{\partial\phi}{\partial y}=\frac{\Gamma}{2\pi}\left[\frac{x-x_0}{(x-x_0)^2+(y-y_0)^2}+\frac{x+x_0}{(x+x_0)^2+(y+y_0)^2}-\frac{x+x_0}{(x+x_0)^2+(y-y_0)^2}-\frac{x-x_0}{(x-x_0)^2+(y+y_0)^2}\right] \tag{3}$$

The vortex velocity components u_0 and v_0 can be obtained from Eq. (2) and (3) by omitting the first terms and setting $x=x_0$ and $y=y_0$; we obtain then

$$u_0=\frac{\partial x_0}{\partial t}=\frac{\Gamma}{4\pi}\left[\frac{x_0^2}{y_0(x_0^2+y_0^2)}\right] \tag{4}$$

$$v_0=\frac{\partial y_0}{\partial t}=\frac{\Gamma}{4\pi}\left[\frac{-y_0^2}{x_0(x_0^2+y_0^2)}\right] \tag{5}$$

Elimination of time in the above equations and integration yields an equation for the path of the vortices:

$$y_0^2=\frac{(\pi s/4)^2 x_0^2}{x_0^2-(\pi s/4)^2} \tag{6}$$

The disturbance pressure field is computed from the relation

$$\nabla p=-\rho\frac{\partial\phi}{\partial t}-\frac{1}{2}\rho[u^2+v^2] \tag{7}$$

in which ρ is the fluid density. Since,

$$\frac{\partial\phi}{\partial t}=\frac{\partial\phi}{\partial x_0}\frac{\partial x_0}{\partial t}+\frac{\partial\phi}{\partial y_0}\frac{\partial y_0}{\partial t} \tag{8}$$

we obtain using the derived equations the expression for the pressure on the ground plane $y=0$:

$$\frac{\Delta p}{\frac{1}{2}\rho V^2}=\left(\frac{4C_L s}{\pi^2 A}\right)^2\left\{\frac{x_0^2(x_0^2+y_0^2+x^2)}{[y_0^2+(x-x_0)^2][y_0^2+(x+x_0)^2](x_0^2+y_0^2)}+\frac{y_0^2(x_0^2+y_0^2-x^2)}{[y_0^2+(x-x_0)^2][y_0^2+(x+x_0)^2](x_0^2+y_0^2)}-\left[\frac{4xx_0y_0}{[y_0^2+(x-x_0)^2][y_0^2+(x+x_0)^2]}\right]^2\right\} \tag{9}$$

which makes use of the relation for elliptically loaded wings

$$\Gamma=\frac{4C_L Vs}{\pi A} \tag{10}$$

where C_L is the wing lift coefficient and A is the wing aspect ratio. The pressure as expressed by Eq. (9) varies in time because the vortex positions $\pm x_0, y_0$, are changing with time. The time and position relationship has been obtained by numerical integration of Eqs. (4) and (5) using Eq. (6).

The pressure distribution at various times in shown graphically in Fig. 1. It is interesting to see that as the vortices first approach the ground, only positive pressures are produced; however, as the descent continues the high velocity field of the vortices makes itself apparent in the dips to subatmospheric pressure which lie closely beneath the vortex center. Nevertheless, a substantial positive pressure hill precedes the path of the vortex as it moves laterally in the "ground effect." The level of pressures produced is dependent only on

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